MATH 300 HW #1: Selected solutions

Section 1.1, # 13

(a) Define the symbol ⓥ by

A	B	$A ⓥ B$
T	T	F
T	F	T
F	T	T
F	F	F

So $A ⓥ B$ is true only when exactly one of A and B is true.

(b) Proof: Consider the truth table

A	B	$(A \vee B) \wedge \sim(A \wedge B)$
T	T	F
T	F	T
F	T	T
F	F	F

Note this agrees with the truth table from part (a). ∎

Section 1.3, #3

(a) $\exists k \in \mathbb{Z},\ ak = b$

(b) $\left(\forall a \in \mathbb{N},\ \left[(\exists k \in \mathbb{Z},\ ak = n) \Rightarrow (a=1) \vee (a=n)\right]\right) \wedge (n > 1)$

(c) $\exists a \in \mathbb{N},\ \left[(a \neq 1) \wedge (a \neq n) \wedge \left(\exists k \in \mathbb{Z},\ ak = n\right)\right]$

(d) $\forall a\ \left((a \in A) \Leftrightarrow (a \in B)\right)$, equivalently: $(\forall a \in A,\ a \in B) \wedge (\forall b \in B,\ b \in A)$

(e) $\forall a \in A,\ a \in B$

(f) $\exists a \in A,\ a \notin B$.

(Note there are often several ways to write the same definition)